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EM Algorithm in Estimating the 2- and 3-Parameter Burr Type III Distributions

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Abstract. The Burr Type III distribution has been applied in the study of income, wage and wealth. It is suitable to fit lifetime data since it has flexible shape and controllable scale parameters. The popularity of Burr Type III distribution increases because it has included the characteristics of other distributions such as logistic and exponential. Burr Type III distribution has two categories: First a two-parameter distribution which has two shape parameters and second a three-parameter distribution which has a scale and two shape parameters. Expectation-maximization (EM) algorithm method is selected in this paper to estimate the two- and three-parameter Burr Type III distributions. Complete and censored data are simulated based on the derivation of *pdf* and *cdf* in parametric form of Burr Type III distributions. Then, the EM estimates are compared with estimates from maximum likelihood estimation (MLE) approach through mean square error. The best approach results in estimates with a higher approximation to the true parameters are determined. The result shows that the EM algorithm estimates perform better than the MLE estimates for two- and three-parameter Burr Type III distributions in the presence of complete and censored data.

Keywords: Burr Type III Distribution, EM Algorithm, Censored Data
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INTRODUCTION

[1] first introduced Burr distribution in the cumulative frequency function. The distribution is in a class of continuous probability distributions for a non-negative random variable which can be adopted as a parametric survival model. Burr Type III distribution is one of the types of Burr distribution and has at least two parameters which include shape, scale and location parameters. Burr Type III distribution is widely known amongst researchers for the purpose of statistical modeling and fitting. [2] stated that in economics, the Burr Type III distribution is known as Dagum distribution, in the actuarial literature, it is called an inverse Burr distribution and in meteorological literature, it is named as kappa distribution. Three different methods which are maximum likelihood estimation (MLE), uniformly minimum-variance unbiased estimator (UMVUE) and Bayes estimator approaches had been used by [3] to estimate the parameter of stress-strength model that was treated as random variable of 2-parameter Burr Type III distribution. Recently, [4] used MLE and Bayesian estimation approaches to estimate the shape parameter of Burr Type III distribution based on doubly censored type II data using MathCAD.

In this work, two- and three-parameter Burr Type III distributions are investigated through the simulation and estimation processes. Simulation is carried out by using inverse cumulative distribution function (*cdf*) since the *cdf* of the distribution is strictly monotonic increasing function. The simulated data are either complete or censored since censoring gives different values of estimated parameters for different censoring level. Nowadays, censoring is extensively used in research field when not all values of measurement are known. In other words, the survival times are not always observed. The MLE and expectation-maximization (EM) algorithm estimation approaches are performed in estimating the distribution parameters. Besides using the complete data, this work also will estimate the 2- and 3-parameter Burr Type III distributions in the presence of censored data with different censoring levels by varying the value of shape and scale parameters.

The estimated parameters are next compared for the cases of complete and censored data with different censoring level based on mean square error (MSE). The best method will result in estimates having small mean square error.

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METHODOLOGY

Estimation of 2-parameter Burr Type III Distribution

Two-parameter Burr Type III distribution involves two shape parameters which are c and k. Since the *cdf* of 2-parameter Burr Type III Distribution is strictly monotonic increasing function (Figure 1 (a)), the inversion method is used to simulate the complete and censored data. The equation becomes

$$F^{-1}(u)=(u^{-\frac{1}{k}}-1)^{-\frac{1}{c}} \tag{1}$$

EM algorithm stands for expectation for E and maximization for M which means that this method involves two steps in estimating the parameter of the distribution. Compared to EM, MLE method is more sensitive to its initial value. The value of estimated parameter is different when the initial value is changed.

First, let $y=(y_1{}^T,\ldots,y_n{}^T)^T$ denote the observed data at time T, where $y_i=(d_i,\delta_i)^T$ and $\delta_i=0$ for censored data or 1 for failure (observe) data. T_i is censored or uncensored at $d_i(i=1,\ldots,n)$. Then, the probability density function of 2-parameter Burr Type III distribution is calculated as follows when given $T>d_j$ where j is stage of censoring

$$f(t\mid t>d_j)=\frac{f(t)}{1-F(d_j)}=ck[1-(1+d_j{}^{-c})^k]\frac{t^{-c-1}}{(1+t^{-c})^{k+1}} \tag{2}$$

E-step involves the computation of Q-function. The Q-function of 2-parameter Burr Type III distribution is computed as follows:

$$\begin{aligned}Q(\theta,\theta^{(m)})&=E_{\theta^{(m)}}[\log\ L_c(c,k)]\\&=n\log k+n\log c-(c+1)\sum_{i=1}^{r}\log(d_i)-(k+1)\sum_{i=1}^{r}\log(1+d_i{}^{-c})-\\&\quad(c+1)\sum_{j=r+1}^{n}E_{\theta^{(m)}}(\log T_j\mid T_j>d_j)-(k+1)\sum_{j=r+1}^{n}E_{\theta^{(m)}}[\log(1+T_j{}^{-c})\mid T_j>d_j]\end{aligned} \tag{3}$$

Taylor series expansion is applied in Q-function since the parameter c exists in the other term and the term cannot be solved straightforwardly using numerical integral.

$$\begin{aligned}E_{\theta^{(m)}}[\log(1+T_j{}^{-c})\mid T_j>d_j]&\cong E_{\theta^{(m)}}[\log(1+T_j{}^{-c})\mid T_j>d_j]+c-c^{(m)}E_{\theta^{(m)}}[\frac{T_j{}^{-c}}{1+T_j{}^{-c}}\log(T_j)\mid T_j>d_j]+\\&\quad\frac{1}{2}(c-c^{(m)})^2E_{\theta^{(m)}}[\frac{T_j{}^{-c}}{(1+T_j{}^{-c})^2}(\log(t_i))^2\mid T_j>d_j]\end{aligned} \tag{4}$$

Then, numerical integral is used to solve the term in Q-function as follows:

$$E_{\theta^{(m)}}(\log T_j\mid T_j>d_j)=ck[1-(1+d_j{}^{-c})^k]\int_{d_i}^{\infty}\frac{(\log t_j)(t_j{}^{-c-1})}{(1+t_j{}^{-c})^{k+1}}dt_j \tag{5}$$

$$E_{\theta^{(m)}}[\log(1+T_j{}^{-c})\mid T_j>d_j]=ck[1-(1+d_j{}^{-c})^k]\int_{d_j}^{\infty}\frac{[\log(1+t_j{}^{-c})](t_j{}^{-c-1})}{(1+t_j{}^{-c})^{k+1}}dt_j \tag{6}$$

$$E_{\theta^{(m)}}[\frac{T_j{}^{-c}}{1+T_j{}^{-c}}\log(T_j)\mid T_j>d_j]=ck[1-(1+d_j{}^{-c})^k]\int_{d_j}^{\infty}\frac{[\log(t_j)](t_j{}^{-2c-1})}{(1+t_j{}^{-c})^{k+2}}dt_j \tag{7}$$

$$E_{\theta^{(m)}}[\frac{T_j{}^{-c}}{(1+T_j{}^{-c})^2}(\log(t_i))^2\mid T_j>d_j]=ck[1-(1+d_j{}^{-c})^k]\int_{d_j}^{\infty}\frac{[\log(t_j)]^2(t_j{}^{-2c-1})}{(1+t_j{}^{-c})^{k+3}}dt_j \tag{8}$$

Estimation of 3-parameter Burr Type III Distribution

Three-parameter Burr Type III distribution involves two shape parameters which are *c* and *k* and one scale parameter, *s*. The *cdf* of 3-parameter Burr Type III is also strictly monotonic increasing function (Figure 1 (b)). Therefore, the equation becomes

$$F^{-1}(u) = \frac{s}{(u^{-\frac{1}{k}} - 1)^{\frac{1}{c}}} \tag{9}$$

As given in above section, the probability density function of 3-parameter Burr Type III distribution is calculated as follows when given $T > d_j$

$$f(t \mid t > d_j) = \frac{f(t)}{1 - F(d_j)} = cks[1 - (1 + (\frac{s}{d_i})^c)^k] \frac{(\frac{s}{t})^{c-1}}{t^2[1 + (\frac{s}{t_j})^c]^{k+1}} \tag{10}$$

The Q-function of 3-parameter Burr Type III distribution is obtained as

$$Q(\theta, \theta^{(m)}) = E_{\theta^{(m)}}[\log L_c(c,k,s)]$$

$$= n\log ck + n\log s + (c-1)\sum_{i=1}^{r}\log(\frac{s}{d_i}) + \sum_{i=1}^{r} d_i^{\,2} - (k+1)\sum_{i=1}^{r}\log(1 + (\frac{s}{d_i})^c) + (c-1)$$

$$\sum_{j=r+1}^{n} E_{\theta^{(m)}}[\log(\frac{s}{T_j}) \mid T_j > d_j] + \sum_{j=r+1}^{n} E_{\theta^{(m)}}[T_j^{\,2} \mid T_j > d_j] - (k+1)\sum_{j=r+1}^{n} E_{\theta^{(m)}}[\log(1 + (\frac{s}{T_j})^c) \mid T_j > d_j] \tag{11}$$

The Q-function is solved using Taylor series expansion as follows

$$E_{\theta^{(m)}}[\log(1 + (\frac{s}{T_j})^c) \mid T_j > d_j] \cong E_{\theta^{(m)}}[\log(1 + (\frac{s}{T_j})^c) \mid T_j > d_j] +$$

$$c - c^{(m)} E_{\theta^{(m)}}[\frac{(\frac{s}{T_j})^c}{1 + (\frac{s}{T_j})^c}\log(\frac{s}{T_j}) \mid T_j > d_j] + \frac{1}{2}(c - c^{(m)})^2 E_{\theta^{(m)}}[\frac{(\frac{s}{T_j})^c}{(1 + (\frac{s}{T_j})^c)^2}(\log(\frac{s}{T_j}))^2 \mid T_j > d_j] \tag{12}$$

Then, numerical integral is used to solve the expected values and the equation becomes

$$E_{\theta^{(m)}}[\log(\frac{s}{T_j}) \mid T_j > d_j] = cks[1 - (1 + (\frac{s}{d_i})^c)^k] \int_{d_i}^{\infty} \frac{[\log(\frac{s}{t_j})][(\frac{s}{t_j})^{c-1}]}{t_j^{\,2}[1 + (\frac{s}{t_j})^c]^{k+1}} dt_j \tag{13}$$

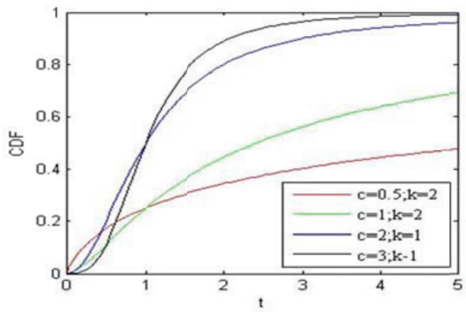
$$E_{\theta^{(m)}}[\log(1 + (\frac{s}{T_j})^c) \mid T_j > d_j] = cks[1 - (1 + (\frac{s}{d_i})^c)^k] \int_{d_i}^{\infty} \frac{[\log[1 + (\frac{s}{t_j})^c]][(\frac{s}{t_j})^{c-1}]}{t_j^{\,2}[1 + (\frac{s}{t_j})^c]^{k+1}} dt_j \tag{14}$$

$$E_{\theta^{(m)}}[\frac{(\frac{s}{T_j})^c}{1 + (\frac{s}{T_j})^c}\log(\frac{s}{T_j}) \mid T_j > d_j] = cks[1 - (1 + (\frac{s}{d_i})^c)^k] \int_{d_i}^{\infty} \frac{\log(\frac{s}{t_j})(\frac{s}{t_j})^{2c-1}}{t_j^{\,2}[1 + (\frac{s}{t_j})^c]^{k+2}} dt_j \tag{15}$$

$$E_{\theta^{(m)}}[\frac{(\frac{s}{T_j})^c}{(1 + (\frac{s}{T_j})^c)^2}(\log(\frac{s}{T_j}))^2 \mid T_j > d_j] = cks[1 - (1 + (\frac{s}{d_i})^c)^k] \int_{d_i}^{\infty} \frac{[\log(\frac{s}{t_j})]^2(\frac{s}{t_j})^{2c-1}}{t_j^{\,2}[1 + (\frac{s}{t_j})^c]^{k+3}} dt_j \tag{16}$$

$$E_{\theta^{(m)}}[T_j^{\,2} \mid T_j > d_j] = cks[1-(1+(\frac{s}{d_i})^c)^k]\int_{d_i}^{\infty}\frac{(\frac{s}{t_j})^{c-1}}{[1+(\frac{s}{t_j})^c]^{k+1}}dt_j \tag{17}$$

The M-step is solved by maximizing the Q-function of 2- and 3-parameter Burr Type III distributions. Last but not least, the E and M steps are repeated until the estimated values converge to the true parameter values of the 2- and 3-parameter Burr Type III distributions.



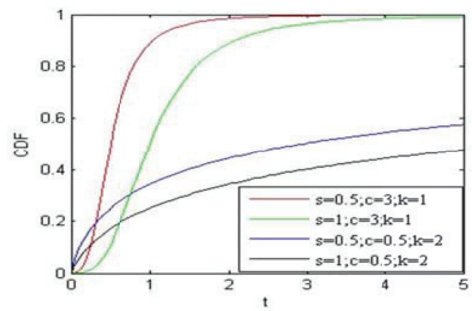


FIGURE 1. (a) *cdf* of 2-parameter Burr Type III distribution **and (b)** *cdf* of 3-parameter Burr Type III distribution

RESULT AND DISCUSSION

2- and 3-parameter of Burr Type III distribution are written as $Burr_{III}$ (c, k) and $Burr_{III}$ (c, k, s) respectively. This work changes the true value of parameter by varying the c values (Table 1) and k values (Table 2). The discussion and explanation is given to $Burr_{III}$ (1, 2) from Table 1 since the explanation is the same for other parameter values of 2-parameter Burr Type III distribution. $Burr_{III}$ (1, 2) from Table 1 shows that for 0% censoring level, the estimated parameter using MLE is 0.9028 for c parameter and 2.0072 for k parameter meanwhile using EM, the estimates are 0.9851 for c and 2.0018 for k. The MSEs of estimated parameter using EM is smaller than using MLE which is 0.0002 for c and 0.0000 for k parameters. The accuracy of the estimators drops in the presence of 5% censoring level with the value 0.7678 for c and 2.1989 for k using MLE. When using EM algorithm, the value raises with 1.2188 and 2.1550 for c and k respectively. 10% censoring level shows that when using EM, the estimated parameter for c is 1.4592 with MSE is 0.2109 and for k is 2.4110 with MSE is 0.1689.

TABLE (1). Comparison of the estimators and MSE (in parentheses) and negative log-likelihood for multiple data sets of 2-parameter Burr Type III distribution with true value of c = 1, 1.5, 3 and k = 2

c	**CL**	**MLE (MSE)**		**-ln L**	**EM (MSE)**		**-ln L**
		c	***k***		***c***	***k***	
1	0%	0.9028 (0.0094)	2.0072 (0.0001)	569.8829	0.9851 (0.0002)	2.0018 (0.0000)	568.3192
	5%	0.7678 (0.0539)	2.1989 (0.0396)	569.3710	1.2188 (0.0479)	2.1550 (0.0240)	568.8156
	10%	0.4817 (0.2686)	2.4965 (0.2465)	639.7209	1.4592 (0.2109)	2.4110 (0.1689)	655.1184
1.5	0%	1.3956 (0.0109)	2.0261 (0.0007)	402.8784	1.5285 (0.0008)	1.9602 (0.0016)	400.9133
	5%	1.3755 (0.0155)	2.1781 (0.0317)	394.7862	1.5993 (0.0099)	2.1840 (0.0339)	392.7608
	10%	0.7608 (0.5464)	2.5304 (0.2813)	487.8320	1.7201 (0.0484)	2.2839 (0.0806)	458.3111
3	0%	2.8347 (0.0273)	2.0089 (0.0001)	229.8279	3.0049 (0.0000)	1.9942 (0.0000)	231.0889
	5%	1.8276 (1.3745)	2.2976 (0.0886)	274.0533	3.2102 (0.0442)	1.7310 (0.0724)	259.7112
	10%	0.8641 (4.5621)	2.6091 (0.3710)	434.2345	3.4578 (0.2096)	1.5742 (0.1813)	389.5635

TABLE (2). Comparison of the estimators and MSE (in parentheses) and negative log-likelihood for multiple data sets of 2-parameter Burr Type III distribution with true value of c = 2.5 and k = 1.5, 2, 4

k	**CL**	**MLE (MSE)**		**-ln L**	**EM (MSE)**		**-ln L**
		c	***k***		***c***	***k***	
1.5	0%	2.6628 (0.0265)	1.9673 (0.2184)	269.5439	2.5472 (0.0022)	1.4767 (0.0005)	275.9502
	5%	1.7892 (0.5052	2.3054 (0.6487)	305.2751	2.0089 (0.2412)	1.6338 (0.0179)	275.9975
	10%	1.6147 (0.7838)	2.5351 (1.0351)	330.0469	1.8578 (0.4124)	1.9189 (0.1755)	277.7179
2	0%	2.5285 (0.0008)	2.0078 (0.0001)	259.8105	2.5049 (0.0000)	2.0025 (0.0000)	259.7451
	5%	2.3295 (0.0291)	2.2157 (0.0465)	264.2935	2.3607 (0.0194)	2.1007 (0.0101)	262.0022
	10%	1.4407 (1.1221)	2.5694 (0.3242)	339.7403	1.7769 (0.5229)	2.4735 (0.2242)	262.0022
4	0%	2.6042 (0.0109)	2.0267 (3.8939)	238.1598	2.5168 (0.0003)	4.0260 (0.0006)	333.6915
	5%	1.5637 (0.8767)	2.3107 (2.8537)	301.2669	1.8552 (0.4158)	4.4737 (0.2244)	369.8970
	10%	1.1027 (1.9524)	2.5323 (2.1541)	370.1766	1.6358 (0.7468)	4.8158 (0.6655)	404.5368

By varying the s values (Table 3), k values (Table 4) and c values (Table 5), the explanation is given to $Burr_{III}$ (2, 4, 1) from Table 4 since the explanation is the same for other parameter values of 3-parameter Burr Type III distribution. The estimators of complete data using MLE are 1.9877 for c, 4.2170 for k and 1.3073 for s. Meanwhile, the estimators using EM are 1.9967 for c, 4.0818 for k and 1.0440 for s. The estimated parameter using EM is closer to the true value of the parameter than the estimated parameter using EM. The MSEs of the estimated parameter using EM is smaller than using MLE with the value 0.0000 for c, 0.0067 for k and 0.0019 for s. The estimated parameter is more far away from the true value in the presence of 5% and 10% censoring level since s is the scale parameter. When censoring is presented in the data, the scale parameter of the distribution changes drastically from the original distribution for both methods. Result of 5% censoring level shows that using EM method give more close value to the true value than using MLE with the value 1.8328 for c, 3.5797 for k and 9.7256 for s with MSEs 0.0280, 0.1767 and 76.1361 for c, k and s respectively. The result is also same for 10% censoring level. The estimators using EM method are 1.4610 with MSE 0.2905 for c, 2.2792 with MSE 2.9612 for k and 17.6416 with MSE 276.9429 for s. The estimated parameter using MLE and EM approaches for both 5% and 10% censoring level of 2- and 3-parameter Burr Type III distribution becomes more far away from the true value and when the number of incomplete data (censored) increases, the value of the negative log-likelihood is higher. The sensitivity of the estimators especially which included higher censoring level is relatively poor compared to the uncensored data. But EM method still provides the better value of estimated parameter than MLE based on MSE. Hence, EM approach outperforms MLE approach in estimating the 2- and 3-parameter Burr Type III distribution.

TABLE (3). Comparison of the estimators and MSE (in parentheses) and negative log-likelihood for multiple data sets of 3-parameter Burr Type III distribution with true value of c = 2, k = 3 and s =1, 3

s	**CL**	**MLE (MSE)**			**-ln L**	**EM (MSE)**			**-ln L**
		c	***k***	***s***		***c***	***k***	***s***	
1	0%	1.9817 (0.0003)	3.1644 (0.0270)	1.0447 (0.0020)	-3286.6000	1.9962 (0.0000)	3.0589 (0.0035)	1.0075 (0.0001)	-3287.2000
	5%	1.7973 (0.0411)	2.6205 (0.1440)	11.0611 (101.2257)	1146	1.8302 (0.0288)	2.7554 (0.0598)	10.5656 (91.5007)	1203.9000
	10%	1.4627 (0.2887)	1.8463 (1.3310)	20.5829 (383.4900)	841.5817	1.6451 (0.1260)	2.0619 (0.8800)	19.2173 (331.8700)	1078.8000
3	0%	2.1846 (0.0341)	2.9336 (0.0044)	3.0133 (0.0002)	-16282	2.0558 (0.0031)	2.9960 (0.0000)	2.9978 (0.0000)	-16281
	5%	1.9311 (0.0047)	2.7287 (0.0736)	5.8430 (8.0826)	-2140.6000	1.9380 (0.0038)	2.7530 (0.0610)	5.2095 (4.8819)	-2176
	10%	1.6423 (0.1279)	2.1974 (0.6442)	10.4694 (55.7919)	-2709	1.7012 (0.0893)	2.2117 (0.6214)	10.1512 (51.1397)	-2709.5000

TABLE (4). Comparison of the estimators and MSE (in parentheses) and negative log-likelihood for multiple data sets of 3-parameter Burr Type III distribution with true value of c = 2, k = 1, 4 and s = 1

k	**CL**	**MLE (MSE)**			**-ln L**	**EM (MSE)**			**-ln L**
		c	***k***	***s***		***c***	***k***	***s***	
1	0%	1.9224 (0.0060)	1.1040 (0.0108)	1.1007 (0.0101)	-966.6940	1.9836 (0.0003)	1.0185 (0.0003)	1.0946 (0.0089)	-967.7521
	5%	1.8749 (0.0157)	1.0160 (0.0003)	1.1229 (0.0151)	228.2781	1.8834 (0.0136)	1.1660 (0.0273)	1.1651 (0.0273)	235.7809
	10%	1.7406 (0.0673)	0.8655 (0.0181)	1.2098 (0.0440)	91.1424	1.7533 (0.0609)	1.2762 (0.0763)	1.2332 (0.0544)	123.6668
4	0%	1.9877 (0.0002)	4.2170 (0.0471)	1.3073 (0.0944)	-4449	1.9967 (0.0000)	4.0818 (0.0067)	1.0440 (0.0019)	-4470.7000
	5%	1.7708 (0.0525)	3.3173 (0.4661)	10.1402 (83.5433)	1204.5000	1.8328 (0.0280)	3.5797 (0.1767)	9.7256 (76.1361)	1333.7000
	10%	1.3974 (0.3631	2.1685 (3.3544)	18.8243 (317.7057)	765.3648	1.4610 (0.2905)	2.2792 (2.9612)	17.6416 (276.9429)	834.4739

TABLE (5). Comparison of the estimators and MSE (in parentheses) and negative log-likelihood for multiple data sets of 3-parameter Burr Type III distribution with true value of c = 1, 4, k = 3 and s =1

c	**CL**	**MLE (MSE)**			**-ln L**	**EM (MSE)**			**-ln L**
		c	***k***	***s***		***c***	***k***	***s***	
1	0%	1.0157 (0.0002)	3.0127 (0.0002)	0.9943 (0.0000)	-1369400	1.0082 (0.0000)	2.9978 (0.0000)	1.0008 (0.0000)	-1369400
	5%	1.0136 (0.0002)	2.7588 (0.0582)	12.9106 (141.8624)	-609750	0.9710 (0.0008)	2.8388 (0.0260)	11.8774 (118.3178)	-609770
	10%	0.9328 (0.0045)	2.2680 (0.5358)	27.3663 (695.1818)	-610520	1.0458 (0.0021)	2.3994 (0.3607)	24.7671 (564.8750)	-610430
4	0%	4.9314 (0.0047)	3.1490 (0.0222)	0.8668 (0.0177)	-448.1894	4.9825 (0.0003)	3.0744 (0.0055)	0.9665 (0.0010)	-474.1326
	5%	4.2100 (0.6241)	2.2527 (0.5585)	0.9201 (0.0064)	4.0849	4.4638 (0.2875)	2.3840 (0.3795)	1.1076 (0.0166)	0.2834
	10%	2.9848 (4.0610)	1.3037 (2.8774)	1.1063 (0.0113)	73.8992	3.5223 (2.1836)	1.5087 (2.2240)	1.1312 (0.0172)	83.1343

CONCLUSION

EM algorithm approach has been used in this work to estimate the 2- and 3-parameter Burr Type III distribution using complete and censored data. The values of estimated parameters are far away from the true parameter values when the censoring level increases. Burr Type III distribution is very sensitive to the censoring level and although the small censoring level is used, the differences of estimated parameters and the true values are quite high. The result of estimated parameters using EM is then compared with those from MLE approach. By performing the estimation process through EM algorithm, the estimated parameters give closer estimated value to the true parameter values of 2- and 3-parameter Burr Type III distributions compared to those from the MLE method. Therefore, EM algorithm method perform better than the MLE estimates for the 2- and 3-parameter Burr Type III distributions for both cases of complete and censored data.

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REFERENCES

1. I. W. Burr, *Annals of Mathematical Statistics* **13,** 215-232 (1942).
2. C. Kleiber, *A Guide to the Dagum Distributions*, Center of Business and Economics (WWZ), University of Basel, 2007
3. A. I. Shawky and F. H. Al-Kashkari, *International Journal of Statistics* **3(LXV)**, 371-385 (2007)
4. A. M. Abd-Elfattah and A. H. Al-Harbey, New York: *ISRN Applied Mathematics* **2012,** 1-18 (2012).