GROUP ASSIGNMENTS (CHAPTER 1)

Group (1)

Question 1

Solve the following equations

a. $\boldsymbol{x\frac{dy}{dx} = \cot y}$,

b. $\boldsymbol{\frac{dy}{dx} + (1 + y^2) = 0}$, $\boldsymbol{y(0) = 0.}$

Question 2

(Bernoulli Equations) The equation

$$\frac{dy}{dx} + 2y = xy^{-2} \tag{1}$$

is an example of a Bernoulli equation.

(a) Show that the substitution $v = y^3$ reduces equation (1) to

$$\frac{dv}{dx} + 6v = 3x. \tag{2}$$

(b) Solve equation (2) for *v*. Then make the substitution $v = y^3$ to obtain the solution to equation (1).

Group (2)

Question 1

Solve the initial value problem

$$e^x dy - y^2 x dx = 0, \quad y(0) = 3.$$

Question 2

An object falls from a high place towards earth with zero initial velocity. The velocity of the object, $v(t)$ satisfies the equation

$$\frac{dv}{dt} = g - kv,$$

where *g* is the acceleration due to gravity and *k* > 0, is a constant. Calculate the time taken when velocity is $\frac{g}{2k}$.

Group (3)

Question 1

Show that the given equation is homogeneous. Hence solve the equation

$$\frac{dy}{dx} = \frac{x^2 + xy + 3y^2}{x^2 + 2xy}.$$

Question 2

Just before midday the body of an apparent homicide victim is found in a room that is kept at a constant temperature of 70°F. At 12 noon, the temperature of the body is 80°F and at 1pm it is 75°F. Assume that the temperature of the body at the time of death was 98.6°F and that it has cooled in accord with Newton's Law. What was the time of death?

Group (4)

Question 1

Show that the equation

$$\left(3x^2 + y\cos x\right)dx + \left(\sin x - 4y^3\right)dy = 0$$

is exact. Hence solve the equation.

Question 2

(Compounded Interest) Upon the birth of their first child, a couple deposited RM5000 in an account that pays 8% interest compounded continuously. The interest payments are allowed to accumulate. How much will the account contain on the child's eighteenth birthday?

Group (5)

Question 1

Solve the linear equation

$$\sin x \frac{dy}{dx} + y\cos x = 2e^x \sin x.$$

Question 2

(Drug elimination) Suppose that sodium pentobarbitol is used to anesthetize a dog. The dog is anesthetized when its bloodstream contains at least 45mg of sodium pentobarbitol per kg of the dog's body weight. Suppose also that sodium pentobarbitol is eliminated exponentially from the dog's bloodstream, with a half-life of 5 hours. What single dose should be administered in order to anesthetize a 50-kg dog for 1 hour?

Group (6)

Question 1

Show that the differential equation

$$x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$

is homogeneous. Hence, solve the equation.

Question 2

(Half-life Radioactive Decay) A breeder reactor converts relatively stable uranium 238 into the isotope plutonium 239. After 15 years, it is determined that 0.043% of the initial amount $\boldsymbol{A_0}$ of plutonium has disintegrated. Find the half-life of this isotope if the rate of disintegration is proportional to the amount remaining.

Group (7)

Question 1

Find the general solution of the Bernoulli equation

$$\frac{dy}{dx} - \frac{y}{x} = 2xy^3.$$

Question 2

A 30-volt electromotive force is applied to an LR series circuit in which the inductance is 0.1 henry and the resistance is 15 ohms. Find the curve $\boldsymbol{i(t)}$ if $\boldsymbol{i(0) = 0}$. Determine the current as $\boldsymbol{t \to \infty}$.

Group (8)

Question 1

Equation $\left(2x^4y\right)dy+(4x^3y^2-x^3)dx=0,$ can be rewritten as a Bernoulli equation,

$$2x\frac{dy}{dx}+4y=\frac{1}{y}.$$

By using the substitution $z=y^2,$ solve this equation.

Question 2

An electromotive force

$$\boldsymbol{E(t)}\begin{cases}\mathbf{120}, & \mathbf{0}\leq \boldsymbol{t}\leq \mathbf{20}\\ \mathbf{0}, & \boldsymbol{t}\geq \mathbf{20}\end{cases}$$

is applied to an LR series circuit in which the inductance is 20 henries and the resistance is 2 ohms. Find the current $\boldsymbol{i(t)}$ if $\boldsymbol{i(0)=0}$.

Group (9)

Question 1

Given the differential equation

$$(x-2y)\ dx+\ (y-2x)\ dy=0.$$

Show that the differential equation is exact. Hence, solve the differential equation by the method of exact equation.

Question 2

(Free Fall) An object falls through the air towards earth. Assuming that only air resistance and gravity are acting on the object, then the velocity *v* satisfies the equation

$$m\frac{dv}{dt}=mg-bv$$

where *m* is the mass, *g* is the acceleration due to gravity, and *b* > 0 is a constant. If *m* = 100kg, *g* = 9:8 m/sec2, *b* = 5 kg/sec, and *v*(0) = 10m/sec, solve for *v*(*t*). What is the limiting (i.e., terminal) velocity of the object?