# UNIVERSITI TEKNOLOGI MALAYSIA
## FAKULTI SAINS
## JABATAN MATEMATIK

Test 1 : SSE 1793 — Session/Sem : 20082009/II

Answer all the questions — Time : 60 minutes

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**1.** Solve the ordinary differential equation,

$$\frac{dy}{dx} = \frac{x + xy^2}{y \csc x}.$$

[5 marks]

**2.** Solve the initial value problem

$$\cos x \frac{dy}{dx} + y \sin x = 2x \cos^2 x, \qquad y(0) = 5.$$

[6 marks]

**3.** Given the differential equation,

$$\left(\frac{3y}{x} + 6x + 2y^2\right) dx - \left(2 - \ln x^3 - 4xy\right) dy = 0.$$

Show that the equation is exact. Hence, solve it.

[7 marks]

**4.** By using substitution $z = y^3$, solve the differential equation

$$\frac{dy}{dx} + 2y = xy^{-2}.$$

[5 marks]

**5.** By using an appropriate substitution, show that the solution of homogeneous equation

$$(xy + x^2)\frac{dy}{dx} = x^2 + y^2$$

is $C - \frac{y}{x} = \ln\left|x\left(\frac{y}{x} - 1\right)^2\right|$ where $C$ is a constant.

[7 marks]

**Solution Test 1 SSE1793 SEM II 20082009**

**Question 1 (5 marks)**

$$\frac{dy}{dx}=\frac{x+xy^2}{y\cos ecx}=\frac{x\left(1+y^2\right)}{y\cos ecx}$$

$$\frac{y}{1+y^2}dy=x\sin xdx$$ K1 - separate

$$\int\frac{y}{1+y^2}dy=\int x\sin xdx$$

| $u=1+y^2$<br>$\frac{du}{2}=ydy$ | sign | $u$ | $dv$ |
|---|---|---|---|
| | $+$ | $x$ | $\sin x$ |
| | $-$ | $1$ | $-\cos x$ |
| | $+$ | $0$ | $-\sin x$ |

K1 K1

$$\frac{1}{2}\int\frac{1}{u}du=\sin x-x\cos x+c$$

$$\frac{1}{2}\ln u=\sin x-x\cos x+c$$

$$\frac{1}{2}\ln\left|1+y^2\right|=\sin x-x\cos x+c$$

J1 J1

**Note : Must have *c*, constant**

**Question 2 (6 marks)**

Solve IVP:

$$\cos x\frac{dy}{dx}+y\sin x=2x\cos^2 x,\qquad y(0)=5$$

$$\frac{dy}{dx}+y\frac{\sin x}{\cos x}=2x\cos x$$

$$p(x)=\frac{\sin x}{\cos x},\qquad q(x)=2x\cos x$$ K1

$$\mu(x)=e^{\int\frac{\sin x}{\cos x}dx}=e^{-\ln|\cos x|}=\sec x=\frac{1}{\cos x}$$ J1

$$\frac{d}{dx}\left[\frac{1}{\cos x}y\right]=\frac{2x\cos x}{\cos x}=2x$$

$$\frac{y}{\cos x}=\int 2xdx$$ K1

$$\frac{y}{\cos x}=x^2+c$$

$$y=y(x)=\cos x\left(x^2+c\right)$$ J1

Applying I.V.

$$y(0)=\cos 0\left(0^2+c\right)=5\Rightarrow c=5$$ K1

$$y=\cos x\left(x^2+5\right)$$ J1

**Question 3 (7 marks)**

$$M(x,y)=\frac{3y}{x}+6x+2y^2, \qquad N(x,y)=\ln x^3+4xy-2$$

$$M_y=\frac{3}{x}+4y, \qquad N_x=\frac{3x^2}{x^3}+4y=\frac{3}{x}+4y$$ K1 – either one

$M_y=N_x$, therefore the equation is exact. J1

$$\text{Let } \frac{\partial u}{\partial x}=M=\frac{3y}{x}+6x+2y^2$$

$$u(x,y)=\int\left(\frac{3y}{x}+6x+2y^2\right)dx+c(y)$$ K1

$$=3y\ln|x|+3x^2+2xy^2+c(y)$$ J1

$$\frac{\partial u}{\partial y}=3\ln|x|+4xy+c'(y)$$ K1

Compare with $N(x,y)$

$$\Rightarrow c'(y)=-2$$

$$c(y)=\int -2dy=-2y+D$$ J1

$$u(x,y)=3y\ln|x|+3x^2+2xy^2-2y+A$$ B1

**Note : Must have constant**

If student choose

$$\frac{\partial u}{\partial y}=N=\ln x^3+4xy-2$$

$$u(x,y)=\int\left(\ln x^3+4xy-2\right)dy+c(x)$$

$$=y\ln x^3+2xy^2-2y+c(x)$$

$$\frac{\partial u}{\partial x}=\frac{3y}{x}+2y^2+c'(x)$$

Compare with $M(x,y)$

$$\Rightarrow c'(x)=6x$$

$$c(y)=\int 6xdx=3x^2+D$$

$$u(x,y)=y\ln x^3+2xy^2-2y+3x^2+A$$

**Question 4 (5 marks)**

$$\frac{dy}{dx}+2y=xy^{-2}\cdots(1)$$

$$z=y^3,\qquad \frac{dz}{dx}=3y^2\frac{dy}{dx}$$ K1

(1) multiply $y^2$ :

$$y^2\frac{dy}{dx}+2y^3=x$$

$$\frac{1}{3}\frac{dz}{dx}+2z=x$$

$$\frac{dz}{dx}+6z=3x,\quad \text{linear}$$ J1

$$p(x)=6,\qquad q(x)=3x$$

$$\mu(x)=e^{\int 6dx}=e^{6x}$$ B1

$$\frac{d}{dx}\left[e^{6x}z\right]=3xe^{6x}$$ K1

$$e^{6x}z=\int 3xe^{6x}dx$$

$$e^{6x}z=\frac{x}{2}e^{6x}-\frac{1}{12}e^{6x}+c$$

$$z=\frac{x}{2}-\frac{1}{12}+ce^{-6x}$$

$$y^3=\frac{x}{2}-\frac{1}{12}+ce^{-6x}$$ J1

| sign | $u$ | $dv$ |
|---|---|---|
| $+$ | $3x$ | $e^{6x}$ |
| $-$ | $3$ | $\frac{e^{6x}}{6}$ |
| $+$ | $0$ | $\frac{e^{6x}}{36}$ |

**Question 5 (7 marks)**

$$\left(xy+x^2\right)\frac{dy}{dx}=x^2+y^2$$

$$\frac{dy}{dx}=\frac{x^2+y^2}{xy+x^2}$$

$$y=xv,\qquad \frac{dy}{dx}=v+x\frac{dv}{dx}$$ B1

$$\frac{dy}{dx}=\frac{x^2+y^2}{xy+x^2}$$

$$v+x\frac{dv}{dx}=\frac{x^2+(xv)^2}{x(xv)+x^2}=\frac{x^2\left(1+v^2\right)}{x^2(v+1)}$$ K1

$$x\frac{dv}{dx}=\frac{1+v^2}{v+1}-v=\frac{1-v}{1+v}$$

$$\frac{1+v}{1-v}dv=\frac{1}{x}dx$$ J1

$$\int\frac{1+v}{1-v}dv=\int\frac{1}{x}dx$$

$$u=1-v,\ -du=dv,\ 1+v=2-u$$ K1

$$\int\frac{u-2}{u}du=\ln x+c$$

$$\int\left(1-\frac{2}{u}\right)du=\ln x+c$$

$$u-2\ln|u|=\ln x+c$$

$$1-v-2\ln|1-v|=\ln x+c$$ J1 – Both are correct

$$C-v=\ln x+2\ln|1-v|,\quad C=1-c$$ K1

$$C-\frac{y}{x}=\ln\left|x\left(1-\frac{y}{x}\right)^2\right|$$

$$C-\frac{y}{x}=\ln\left|x\left(\frac{y}{x}-1\right)^2\right|$$ J1