UNIVERSITI TEKNOLOGI MALAYSIA
SSCE 1693 ENGINEERING MATHEMATICS I
TUTORIAL 2

1. Differentiate the following functions with respect to x.

   (a) $\tan^2 x$. (b) $\sec(1-3x)$. (c) $\tan^2(1-x)$.

   (d) $\sqrt{\cot x}$. (e) $\sin x \tan x$. (f) $x \operatorname{cosec} 2x$.

2. Differentiate the following functions with respect to x.

   (a) e^{5x}. (b) $e^{\sqrt{x}}$. (c) $e^{\cos x}$.

   (d) $e^{\tan x}$. (e) xe^x. (f) $e^x \sin x$.

3. Find the derivatives of the following functions.

   (a) $\ln(x^3+1)$. (b) $\ln \operatorname{cosec} x$. (c) $\ln(x^n+1)$.

   (d) $\ln(x+1)(2x+1)$. (e) $\log_{10} x^2$. (f) $\log_a x^3$.

4. (a) If $y = \dfrac{(3+x)(2-x)^2}{(4-3x)^3}$, find the values of x when $\dfrac{dy}{dx} = 0$.

   (b) If $y = (1+x)^2 \ln(1+x)$, find $\dfrac{dy}{dx}$.

5. If $y = (1+x)\sqrt{25-x^2}$, find the values of $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ when $x = 3$.

6. (a) If $y = e^{2x}\cos x$, show that $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 5y = 0$.

   (b) If $y = \dfrac{e^x}{1+x^2}$, show that
   $$(1+x^2)\frac{dy}{dx} = y(x-1)^2.$$

7. (a) If $y = \sin(e^x - 1)$, show that
   $$\frac{d^2y}{dx^2} - \frac{dy}{dx} + ye^{2x} = 0.$$

   (b) If $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$, show that $\dfrac{dy}{dx} = 1 - y^2$.

8. By using logarithm, find the derivatives of the following functions.

   (a) $\dfrac{(1+2x)\sqrt{1+x}}{1-x}$. (b) $\dfrac{(x+1)^2}{(x+2)^2(x+3)^4}$.

(c) $\dfrac{(x+1)^2(x+2)}{(x+3)^3}$. (d) $xe^{ax}\sin bx$.

9. (a) Differentiate the following functions with respect to x.

(i) $\ln(1-\tan^2 x)$. (ii) x^x. (iii) $x^x + x^{\frac{1}{x}}$.

(b) If $y = \dfrac{A+\ln(1+x)}{x^2}$, where A constant, show that

$$x(1+x)\left(x\frac{dy}{dx}+2y\right) = 1.$$

10. (a) If $y = (1+x)\sqrt{25-x^2}$, find the values of $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ where $x = 3$.

(b) If $y = \sec x$, show that

$$\frac{d^2y}{dx^2} = 2\sec^3 x - \sec x.$$

11. (a) If $y = e^{ax}\sin x$, where a is a constant, show that

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2+1)y = 0.$$

Hence, evaluate the first five derivatives of y when $x = 0$.

(b) If $y = (A+Bx)e^{-2x}$, where A and B are constants, show that

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0.$$

12. Find $\dfrac{dy}{dx}$ in terms of t for the following cases.

(a) $x = t^2$, $y = t + \dfrac{1}{t}$. (b) $x = \cos 2t$, $y = 4\sin t + 3$.

(c) $x = \sec t$, $y = \tan t$. (d) $x = \sin^3 t$, $y = t\sin t + \cos t$.

13. If $x = a\cos^3 t$ and $y = a\sin^3 t$, where a is a constant, find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ in terms of t.

14. Find $\dfrac{d^2y}{dx^2}$ when

(a) $x^2 + xy + y^2 = 1$. (b) $x = \sec t$ and $y = \tan t$.

15. Differentiate the given expressions with respect to x.

(a) $\sin^{-1}(3x+2)$. (b) $\tan^{-1}\left(\dfrac{4\sqrt{x}}{1-4x}\right)$. (c) $\cos^{-1}\left(2x\sqrt{1-x^2}\right)$.

(d) $x\cos^{-1}\left(\sqrt{4x+1}\right)$. (e) $\sin^{-1}\left(\dfrac{x^2}{a^2+x^2}\right)$, $a \neq 0$. (f) $\tan^{-1}\left(\dfrac{\cos x - \sin x}{\cos x + \sin x}\right)$.

16. Differentiate the given expressions with respect to x.

(a) $x^2\tan^{-1}\left(\dfrac{x}{2}\right)$. (b) $e^x \sec^{-1} x$. (c) $x^2\left(\sin^{-1} x\right)^3$.

(d) $\sec^{-1}\left(\sqrt{x^2-1}\right)$. (e) $\dfrac{\cos^{-1} x}{x}$. (f) $\dfrac{\tan^{-1} x}{x^2+1}$.

17. If $y = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that

(a) $(1-x^2)\dfrac{dy}{dx} - xy = 1$. (b) $(1-x^2)\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} - y = 0$.

18. Show that $y = \tan^{-1} x$ satisfies the equation

$$\frac{d^2y}{dx^2} + 2\sin y \cos^3 y = 0.$$

19. If $y = \tan^{-1}(\sinh x)$, show that

$$\frac{d^2y}{dx^2} + \tanh x \left(\frac{dy}{dx}\right)^2 = 0.$$

20. If $y = \sqrt{1-x^2}\,\sin^{-1} x$, show that

$$(1-x^2)\frac{dy}{dx} + xy = 1 - x^2.$$

21. (a) If $y = (\sin^{-1} x)^2$, show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2.$$

(b) If $y = \sin^{-1} x + (\sin^{-1} x)^2$, show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$$

is independent of x.

22. If $y = \cosh(\sin^{-1} x)$, show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0.$$

23. If $y = \sinh(m \sinh^{-1} x)$, show that

$$(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2 y = 0.$$

24. If $y = (\cosh^{-1} x)^2$, show that

(a) $(x^2-1)\left(\dfrac{dy}{dx}\right)^2 = 4y.$ (b) $(x^2-1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = 2.$

25. Find $\dfrac{dy}{dx}$ for the given functions by differentiating implicitly.

(a) $\ln(x+y) = \tan^{-1}(xy).$ (b) $\tan^{-1} y = \sin^{-1} x.$

(c) $x^3 + x\tan^{-1} y = e^y.$ (d) $\sin^{-1}(xy) = \cos^{-1}(x-y).$

26. Show that

(a) $\dfrac{d}{dx}\left[\tanh^{-1}\sqrt{2-x}\right] = -\dfrac{1}{2(x-1)\sqrt{2-x}}.$

(b) $\dfrac{d}{dx}\left[\sinh^{-1}(\tan x)\right] = \sec x.$

(c) $\dfrac{d}{dx}\left[\sinh^{-1}(\sin x)\right] = \dfrac{1}{\sqrt{\sec^2 x + \tan^2 x}}.$

27. Differentiate with respect to x

$$\tanh^{-1}\left(\frac{2\sqrt{x}}{1+x}\right) + \tan^{-1}\left(\frac{2\sqrt{x}}{1+x}\right)$$

and simplify your answer.

28. If $y\sinh^{-1} x = x\sinh^{-1} y$, find $\dfrac{dy}{dx}$ in terms of x and y.

29. Find $\dfrac{dy}{dx}$ for the following equations.

(a) $y = \sinh^{-1}(\ln\sinh x).$ (b) $y = \tan^{-1}(\sinh x).$