UNIVERSITI TEKNOLOGI MALAYSIA
SSCE 1693 ENGINEERING MATHEMATICS I
TUTORIAL 7: MATRIX ALGEBRA

1. Calculate the determinant of the following matrices.

   (a) $\begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix}$.   (b) $\begin{vmatrix} 2 & -4 & 3 \\ 3 & 1 & 2 \\ 1 & 4 & -1 \end{vmatrix}$.

   (c) $\begin{vmatrix} 2 & 3 & -4 \\ 4 & 0 & 5 \\ 5 & 1 & 6 \end{vmatrix}$.   (d) $\begin{vmatrix} 4 & 3 & 0 \\ 6 & 5 & 2 \\ 9 & 7 & 3 \end{vmatrix}$.

2. Given that

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 5 & 5 & 0 \\ 0 & 3 & p \\ 0 & q & r \end{pmatrix}.$$

   Calculate AB and BA. Hence, find the values of p, q, and r if $AB = BA$.

3. Given that

$$M = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ 3 & 1 & 1 \end{pmatrix}.$$

   Show that

$$M^3 = M^2 + 6M + I,$$

   where I is an identity matrix. Deduce that

$$M^{-1} = M^2 - M - 6I,$$

   and hence find M^{-1}.

4. Given that

$$P = \begin{pmatrix} -1 & 2 & -3 \\ 4 & 1 & 5 \\ 2 & -1 & 6 \end{pmatrix} \quad \text{and} \quad Q = \begin{pmatrix} -11 & 9 & -13 \\ 14 & 0 & 7 \\ 6 & -3 & 9 \end{pmatrix}.$$

   Calculate PQ. Hence, deduce the inverse matrix of P.

5. Find the values of k for the following equations.

   (a) $\begin{vmatrix} k & 5 & 3 \\ 5 & k+1 & 1 \\ -3 & -4 & k-2 \end{vmatrix} = 0.$ (b) $\begin{vmatrix} 5 & k & 3 \\ k+2 & 2 & 1 \\ -3 & 2 & k \end{vmatrix} = 0.$

6. Given that

$$A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 3 & -4 \\ 3 & -2 & 4 \\ 2 & 1 & 0 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 7 & 8 & -12 \\ -5 & -1 & 7 \\ 1 & -2 & 3 \end{pmatrix}.$$

Calculate AB and AC. Hence, determine the inverse matrix of A.

7. Calculate the inverse of the following matrices by using

(i) elementary row operations method,

(ii) adjoint method.

(a) $\begin{pmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ -4 & 2 & 5 \end{pmatrix}.$ (b) $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix}.$

(c) $\begin{pmatrix} -3 & -1 & 6 \\ 2 & 1 & -4 \\ -5 & -2 & 11 \end{pmatrix}.$ (d) $\begin{pmatrix} -4 & 3 & 4 \\ 12 & -9 & -11 \\ -1 & 1 & 2 \end{pmatrix}.$

(e) $\begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & -4 \\ -5 & 2 & 1 \end{pmatrix}.$ (f) $\begin{pmatrix} -3 & 1 & 2 \\ 2 & 3 & 0 \\ -1 & 1 & 1 \end{pmatrix}.$

8. Given that

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}.$$

Show that $A^2 - 4A - 5I = 0$. Hence, determine the inverse matrix of A.

9. Solve the following system linear equations by using

(i) inverse matrix method,

(ii) Cramer's rule.

(a)
$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 11, \\ 2x_1 - 2x_2 - 3x_3 &= 5, \\ 3x_1 - 5x_2 + 2x_3 &= -3. \end{aligned}$$

(b)
$$\begin{aligned} x_1 - x_2 - 3x_3 &= 2, \\ x_1 - 3x_2 - 13x_3 &= 14, \\ -3x_1 - 4x_2 + 4x_3 &= 0. \end{aligned}$$

(c)
$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= -4, \\ 2x_1 - x_2 + x_3 &= 3, \\ 3x_1 + 2x_2 + x_3 &= 10. \end{aligned}$$

(d)
$$\begin{aligned} 3x_1 + 4x_2 - 5x_3 &= -2, \\ 2x_1 - x_2 - x_3 &= -9, \\ 4x_1 \qquad - 3x_3 &= -13. \end{aligned}$$

10. Determine whether the following system of linear equations has a trivial solution.

(a)
$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0,\\ 2x_2 + 2x_3 &= 0,\\ x_1 + 2x_2 + 3x_3 &= 0. \end{aligned}$$

(b)
$$\begin{aligned} 2x_1 + x_2 - x_3 &= 0,\\ x_1 - 2x_2 - 3x_3 &= 0,\\ -3x_1 - x_2 + 2x_3 &= 0. \end{aligned}$$

(c)
$$\begin{aligned} 3x_1 + x_2 + 3x_3 &= 0,\\ -2x_1 + 2x_2 - 4x_3 &= 0,\\ 2x_1 - 3x_2 + 5x_3 &= 0. \end{aligned}$$

(d)
$$\begin{aligned} x_1 + 2x_2 + x_3 &= 0,\\ -x_1 - 2x_2 - 2x_3 &= 0,\\ -x_1 - x_2 &= 0. \end{aligned}$$

11. Solve the following system of linear equations by using

(i) Gauss elimination method,

(ii) Gauss–Jordan elimination method.

(a)
$$\begin{aligned} 2x_1 - 4x_2 &= 3,\\ -6x_1 + 12x_2 &= -9,\\ 4x_1 - 8x_2 &= 6. \end{aligned}$$

(b)
$$\begin{aligned} x_1 + 2x_2 &= -7,\\ -x_1 - x_2 &= 1,\\ 2x_1 + x_2 &= 5. \end{aligned}$$

Could you solve these system of linear equations by using inverse matrix method or Cramer's rule?

12. Solve the following system of linear equations by using

(i) Gauss elimination method, (ii) Gauss–Jordan elimination method.

(a)
$$\begin{aligned} x_1 + x_2 + x_3 &= -2,\\ -x_1 + x_2 &= 0,\\ x_2 + x_3 &= -1. \end{aligned}$$

(b)
$$\begin{aligned} x_1 + 3x_2 &= 19,\\ x_2 + 3x_3 &= 10,\\ 3x_1 + x_3 &= -5. \end{aligned}$$

(c)
$$\begin{aligned} 4x_1 + 5x_2 + 6x_3 &= 24,\\ 2x_1 + 7x_2 + 12x_3 &= 40,\\ x_1 + 2x_2 + 3x_3 &= 6. \end{aligned}$$

(d)
$$\begin{aligned} x_1 + 2x_2 + 2x_3 &= 9,\\ 2x_1 + 5x_2 - 2x_3 &= 14,\\ x_1 + 3x_2 - 4x_3 &= 5. \end{aligned}$$

13. Solve the following system of linear equations by using

(i) Gauss elimination method, (ii) Gauss–Jordan elimination method.

(a)
$$\begin{aligned} 5\sqrt{x} - \frac{3}{y} + 2z^2 &= 13,\\ 4\sqrt{x} - \frac{2}{y} + 5z^2 &= 13,\\ 2\sqrt{x} + \frac{4}{y} - 3z^2 &= -9. \end{aligned}$$

(b)
$$\begin{aligned} \frac{2}{x} + \frac{1}{y} - \frac{1}{z} &= 8,\\ \frac{3}{x} + \frac{2}{y} &= 11,\\ \frac{3}{y} + \frac{1}{z} &= 10. \end{aligned}$$

(c)
$$\frac{2}{x}+\frac{1}{y}-\frac{2}{z}=10,\quad \frac{6}{x}+\frac{4}{y}+\frac{4}{z}=2,\quad \frac{10}{x}+\frac{8}{y}+\frac{6}{z}=8.$$

(d)
$$\frac{1}{x}+\frac{3}{y}+\frac{4}{z}=8,\quad \frac{2}{x}+\frac{9}{y}+\frac{6}{z}=27,\quad \frac{1}{x}+\frac{5}{y}+\frac{6}{z}=15.$$

14. Show that each of the following matrix has distinct roots of characteristic equation.

(a) $\begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 2 \end{pmatrix}$

Hence find all the eigenvectors.

15. Determine whether W is a subspace of V.

(a) $V=\mathbb{R}^3, \quad W=\left\{\begin{pmatrix} a \\ -a \\ 2a \end{pmatrix}\right\}$

(b) $V=M_{2\times 2}, \quad W=\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad \geq bc\right\}$

16. Let $A=\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$. Determine whether $C=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is in span (A, B).

17. Determine the eigenvalues of the matrix $A=\begin{pmatrix} 3 & -3 & 6 \\ 0 & 2 & -8 \\ 0 & 0 & -2 \end{pmatrix}$

(a) Show that $\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector of A.

(b) Show that $\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$ is also an eigenvector of $B=\begin{pmatrix} 7 & -6 & 2 \\ 1 & 2 & 3 \\ 1 & -3 & 2 \end{pmatrix}$ and write down the corresponding eigenvalues.

(c) Hence, or otherwise, write down an eigenvector of matrix AB and state the corresponding eigenvalue.