— SKEMA —

Department of Mathematical Sciences
Universiti Teknologi Malaysia
Semester II, 20016/17

SSCE1993 **Test 2 (15%)** **1hr. 30mins.**

Answer all questions.

1. Find the mass of the plane region bounded by the graphs $x = y^2$, $y = x+3$, $y = -3$ and $y = 2$ if the density is constant, $\delta(x, y) = 3$.

[6 marks]

2. The line $y = x$ divides the circle $(x-1)^2 + y^2 = 1$ into two regions. Find the area of the smaller region using polar coordinates.

[6 marks]

3. Find the volume of a solid bounded above by the paraboloid $z = 4 - x^2 - y^2$ and below by the plane $z = -2$, and on the side by the cylinder $x^2 + y^2 = 4$.

[7 marks]

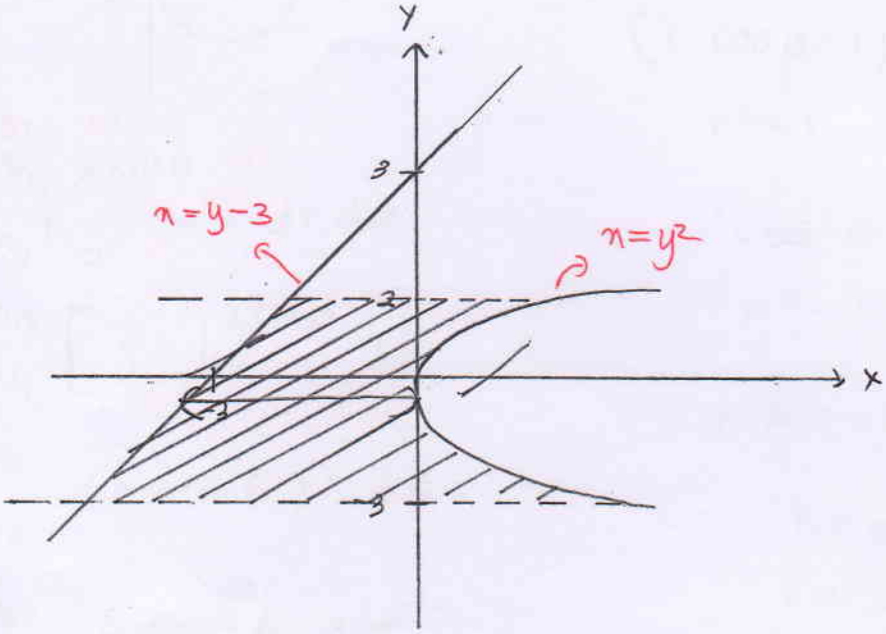
4. Evaluate the following using spherical coordinates:

$$\int_{-\sqrt{2}}^{0} \int_{0}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{2} \left(x^2 + y^2 + z^2\right) dz\, dx\, dy.$$

[7 marks]

Find the mass of the plane region bounded by the graphs

$x=y^2$, $y=x+3$, $y=-3$, $y=2$ when density constant



B1

$$m=\iint_R \left(\delta(x,y) = 3\cancel{c)\,dA}\right.$$

B1 B1

$$=\int_{-3}^{2}\int_{y-3}^{y^2} 3\,dx\,dy$$

$$=\int_{-3}^{2}\Big[3x\Big]_{y-3}^{y^2}dy$$ M1

$$=\int_{-3}^{2}(3y^2-3y+9)\,dy$$

$$=\left[y^3-\frac{3y^2}{2}+9y\right]_{-3}^{2}$$ A1

$$=(8-6+18)-\left(-27-\frac{27}{2}-27\right)$$

$$=20+\frac{135}{2}=\frac{175}{2} \text{ or } 87.5$$ A1

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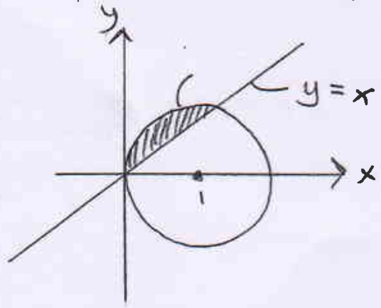
/6

Q2 $y = x \quad (x-1)^2 + y^2 = 1$

Find the area of small region



B1 B1 B1

$$A = \int_{\pi/4}^{\pi/2} \int_0^{2\cos\theta} 1 \cdot r \, dr \, d\theta$$

$$\int_{\pi/4}^{\pi/2} \left[\frac{r^2}{2} \right]_0^{2\cos\theta} d\theta$$

M1

$$\int_{\pi/4}^{\pi/2} (2\cos\theta)^2 \, d\theta$$

$$\int_{\pi/4}^{\pi/2} 4\cos^2\theta \, d\theta$$

$$\cancel{4\left[\sin x \cos x + x\right]_{\pi/4}^{\pi/2}}$$

$$\cancel{= 4(44.5)}$$

$$\cancel{= 178}$$

$$(x-1)^2 + y^2 = 1$$

$$(r\cos\theta - 1)^2 + r^2\sin^2\theta = 1$$

$$r^2 = 1$$

$$r^2\cos^2\theta - 2r\cos\theta = 0$$

$$r^2 - 2r\cos\theta = 0$$

$$r(r - 2\cos\theta) = 0$$

$$r = 0$$

$$r = 2\cos\theta$$

$$4\int_{\pi/4}^{\pi/2} \cos^2\theta \, d\theta$$

$$= 2\int_{\pi/4}^{\pi/2} \left(\frac{1 + \cos 2\theta}{2} \right) d\theta$$

M1

$$= 2\left[\theta + \frac{\sin 2\theta}{2} \right]_{\pi/4}^{\pi/2}$$

$$= 2\left[\left(\frac{\pi}{2} + \frac{\sin \pi}{2} \right) - \left(\frac{\pi}{4} + \frac{\sin(\pi/2)}{2} \right) \right]$$

$$= 2\left(\frac{\pi}{4} - \frac{1}{2} \right)$$

$$= \frac{\pi}{2} - 1 \quad \#$$

A1

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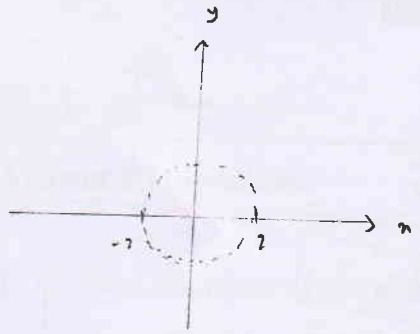
2. $z = 4 - x^2 - y^2$, $z = -2$, $x^2 + y^2 = 4$



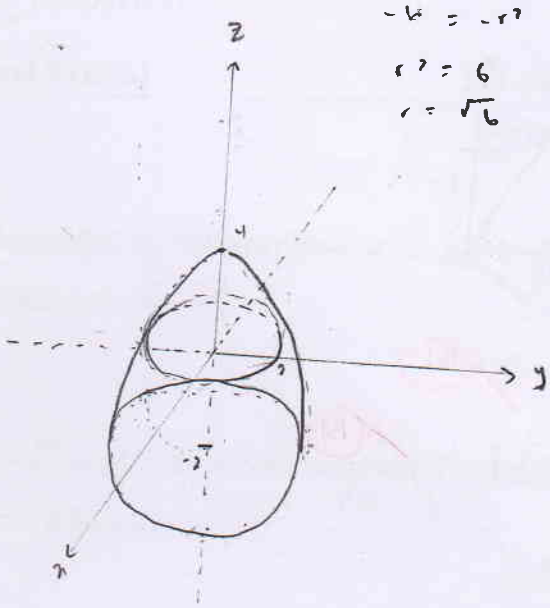
$z = 4 - r^2$

$-2 = 4 - r^2$

$-6 = -r^2$

$r^2 = 6$

$r = \sqrt{6}$



$V = \iiint dV$

$= \int_0^{2\pi} \int_0^{2} \int_{-2}^{4-r^2} r \, dz \, dr \, d\theta$ B1 B1 B1 B1

$= \int_0^{2\pi} \int_0^{\sqrt{6}} r \left[z \right]_{-2}^{4-r^2} dr \, d\theta$

$= \int_0^{2\pi} \int_0^{\sqrt{6}} r \left[(4 - r^2) - (-2) \right] dr \, d\theta$

$= \int_0^{2\pi} \int_0^{\sqrt{6}} r \left[6 - r^2 \right] dr \, d\theta$ M1

$= \int_0^{2\pi} \int_0^{\sqrt{6}} 6r - r^3 \, dr \, d\theta$

$= \int_0^{2\pi} \left[\frac{\cancel{6}^{3} r^2}{\cancel{2}} - \frac{r^4}{4} \right]_0^{\sqrt{6}} d\theta$

$= \int_0^{2\pi} \left[3r^2 - \frac{r^4}{4} \right]_0^{\sqrt{6}} d\theta$ M1

$= \int_0^{2\pi} \left[\left(3(\sqrt{6})^2 - \frac{(\sqrt{6})^4}{4} \right) - 0 \right] d\theta$

$= \int_0^{2\pi} \left[18 - 9 \right] d\theta$

$= 9 \left[\theta \right]_0^{2\pi}$

$= 18\pi$ A1

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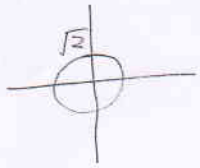
Q4 $\int_{0}^{\sqrt{2}} \int_{0}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2+y^2+z^2)\, dz\, dx\, dy$

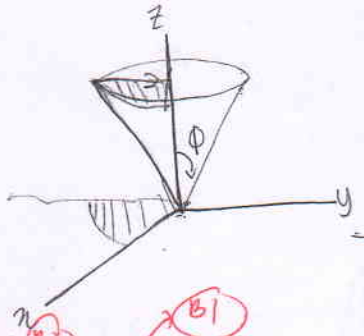
$\rho^2 \, dV$

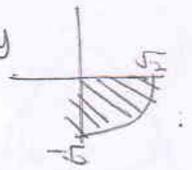
$x^2+y^2=2$

$r^2=2$

$r=\sqrt{2}$







B1 B1 B1 B1

$\int_{0}^{2\pi} \int_{0}^{\frac{\pi}{4}} \int_{0}^{\frac{2}{\cos\phi}} \rho^2 \, dV$

$\frac{\pi}{2}$

$\boxed{z=2}$

$\rho \cos\phi = 2$

$\rho = \frac{2}{\cos\phi}$

$u = \cos\phi$

$\frac{du}{d\phi} = -\sin\phi$

M1 $d\phi = \frac{1}{-\sin\phi}\, du$

$\int_{0}^{2\pi} \int_{0}^{\frac{\pi}{4}} \int_{0}^{\frac{2}{\cos\phi}} \rho^2 \cdot \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$

$\int_{0}^{2\pi} \int_{0}^{\frac{\pi}{4}} \left[\frac{\rho^5}{5}\right]_{0}^{\frac{2}{\cos\phi}} \sin\phi \, d\phi\, d\theta$

$\frac{1}{5}\int_{0}^{2\pi} \int_{0}^{\frac{\pi}{4}} \left(\frac{2}{\cos\phi}\right)^5 \sin\phi \, d\phi\, d\theta$

$\frac{1}{5}\int_{0}^{2\pi} \int_{0}^{\frac{\pi}{4}} \frac{32}{\cos^5\phi} \sin\phi \, d\phi\, d\theta$

$\frac{1}{5}\int_{0}^{2\pi} \int_{0}^{\frac{\pi}{4}} \frac{32}{u^5} \cancel{\sin\phi} \left(\frac{1}{-\cancel{\sin\phi}}\right) du\, d\theta$

A1

$\frac{1}{5}\int_{0}^{2\pi} \int_{0}^{\frac{\pi}{4}} -\frac{1}{u^5}(32)\, du\, d\theta$

$\frac{32}{5}\int_{0}^{2\pi} \left[-\frac{u^{-4}}{(-4)}\right]_{0}^{\frac{\pi}{4}} d\theta$

$\frac{32}{5}\left(\frac{1}{4}\right)\int_{0}^{2\pi} \left[u^{-4}\right]_{0}^{\frac{\pi}{4}} d\theta$

$\frac{8}{5}\int_{0}^{2\pi} \left[\frac{1}{u^4}\right]_{0}^{\frac{\pi}{4}} d\theta$

$\frac{8}{5}\int_{0}^{2\pi} \left(\frac{1}{(\cos\phi)^4} - \frac{1}{(\cos\phi)^4}\right) d\theta$

$\frac{8}{5}\int_{0}^{2\pi} \left(\frac{1}{(\cos\frac{\pi}{4})^4} - \frac{1}{(\cos 0)^4}\right) d\theta$

$\frac{8}{5}\int_{0}^{2\pi} 3 \, d\theta$

$\frac{8}{5}\left[3\theta\right]_{\cancel{0}\,\pi/2}^{2\pi}$

$\frac{8}{5}\left[(3\times 2\pi) - 3(\pi/2)\right]$

$\frac{48}{5}\left(\frac{9}{2}\pi\right) = \frac{36}{5}\pi$ A1

$\cancel{\frac{48}{5}\pi}$

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