# Reliability

SMN 4842 20102011 Project Mgmt and Maintenance Eng.

# Definition of Reliability

"Probability that a system or product will perform in a satisfactory manner for a given period of time when used under specified operating condition"

#### Reliability – 4 main elements

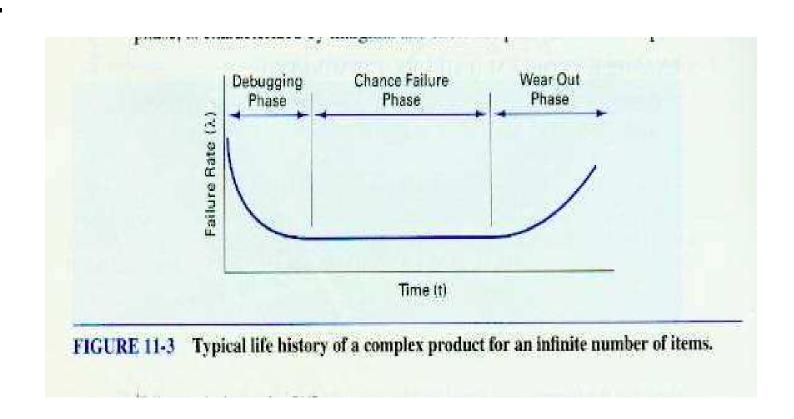
- probability numerical representation number of times that an event occurs (success) divided by total number trials
- Satisfactory performance criteria established which describe what is considered to be satisfactory system operation

- Specifed time measure against which degree of system performance can be related - used to predict probability of an item surviving without failure for a designated period of time
- 4. Specified operating conditions expect a system to function environmental factors, humidity, vibration, shock, temperature cycle, operational profile, etc.

#### LIFE CYCLE CURVE

- typical life history curve for infinite no of items – 'bathtub curve
- comparison of failure rate with time
- 3 distinct phase debugging, chance failure and wear-out phase

# Life Cycle Curve



#### Debugging (Infant mortality) Phase

- rapid decrease in failure rate
- Weibull distribution with shape parameter  $\beta < 1$  is used to describe the occurrences of failure
- Usually covered by warranty period

# Chance failure phase

- Constant failure rate failure occur in random manner
- Exponential and also Weibull with  $\beta=1$  can be used to describe this phase

## Wear-out phase

- Sharp rise in failure rate fatigue, corrosion (old age)
- Normal distribution is one that best describes this phase
- Also can use Weibull with shape parameter  $\beta > 1$

# Maintainability

- Pertains to the ease, accuracy, safety and economy in the performance of maintenance actions
- Ability of an item to be maintained
- Maintainability is a design parameter, maintenance is a result of design

# Measures of Maintainability

- MTBM mean time between maintenance, include preventive and corrective maintenance
- MTBR mean time between replacement, generate spare part requirement
- M mean active maintenance time
- $M_{\rm ct}$  mean corrective maintenance time or mean time to repair
- $\overline{M}_{\rm pt}$  mean preventive maintenance time

Frequency of maintenance for a given time is highly dependent on the reliability of that item

Reliability frequency of maintenance

 Unreliable system require extensive maintenance

# Reliability function [R(t)]

- R(t) = 1 F(t)
- F(t) = probability of a system will fail by time (t) = failure distribution function Eg. If probability of failure F(t) is 20%, then reliability at time t is

$$R(t) = 1 - 0.20 = 0.80 \text{ or } 80\%$$

# Reliability at time (t)

$$ightharpoonup R(t) = e^{-t/\theta}$$

$$e = 2.7183$$

$$\theta = MTBF$$

$$\lambda = \frac{1}{\theta}$$
  $\lambda = \text{failure rate}$ 

So,

$$R(t) = e^{-\lambda t}$$

# Failure Rate $(\lambda)$

Rate at which failure occur in a specified time interval

 $\lambda$ = number of failures

total operating hours

Can be expected in terms of failures per hour, % of failure per 1,000 hours or failures per million hours

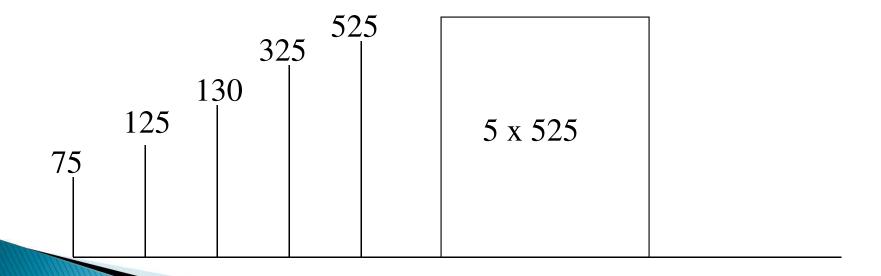
# Example 1

- 10 components were tested. The components (not repairable) failed as follows:
- √ Component 1 failed after 75 ours
- Component 2 failed after 125 hours
- Component 3 failed after 130 hours
- Component 4 failed after 325 hours
- Component 5 failed after 525 hours

#### Determine the MTBF

Solution:

Five failures, operating time = 3805 hours



## Solution

$$\lambda = 5 / 3805 = 0.001314$$

# Example 2

The chart below shows operating time and breakdown time of a machine.



a) Determine the MTBF.

#### Solution:

Total operating time = 
$$20.2 + 6.1 + 24.4 + 4.2 + 35.3 + 46.7$$
  
=  $136.9$  hours

#### Solution

$$\lambda = 4 \ / \ 136.9 = 0.02922$$
 Therefore;  $\theta = MTBF = 1 \ / \ \lambda = 34.22 \ hours$ 

b) What is the system reliability for a mission time of 20 hours?

$$R = e^{-\lambda t} \quad t = 20 \text{ hours}$$
 
$$R = e^{-(0.02922)(20)}$$
 
$$R = 55.74\%$$

# Reliability Component Relationship

 Application in series network, parallel and combination of both

#### Series Network

Most commonly used and the simplest to analyze



All components must operate if the system is to function properly.

$$R = R_A \times R_B \times R_C$$

If the series is expected to operate for a specified time period, then

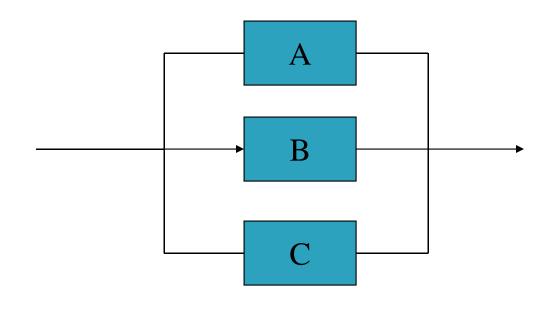
$$\mathbf{R}_{s}(t) = e^{-(\lambda_{1} + \lambda_{2} + \lambda_{3} + \dots + \lambda_{n})t}$$

# Example

Systems expected to operate for 1000 hours. It consists of 4 subsystems in series,  $MTBF_A = 6000$  hours,  $MTBF_R = 4500$  hours,  $MTBF_C = 10,500 \text{ hours}, MTBF_D = 3200$ hours. Determine overall reliability.  $\lambda_A = 1 / MTBF_A = 1/6000 = 0.000167$  $\lambda_{R} = 1/MTBF_{R} = 1/4500 = 0.000222$  $\lambda_{\rm C} = 1/{\rm MTBF_{\rm C}} = 1/10500 = 0.000095$  $\lambda_D = 1/MTBF_D = 1/3200 = 0.000313$ Therefore:  $R = e^{-(0.000797)(1000)} = 0.4507$ 

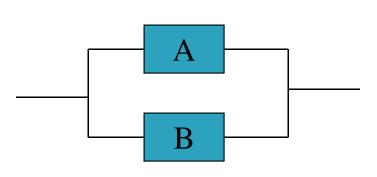
#### Parallel Network

A number of the same components must fail order to cause total system failure



# Example

Consider two units A and B in parallel. The systems fails only when A and B failed.



$$F_{s}(t) = F_{a}(t) F_{b}(t)$$

$$= [1-R_{a}(t)][1-R_{b}(t)]$$

$$= 1-R_{a}(t) R_{b}(t) + R_{a}(t) R_{b}(t)$$

$$R_{s}(t) = 1-F_{s}(t)$$

$$= R_{a}(t) + R_{b}(t) - R_{a}(t) R_{b}(t)$$

If A and B are constant failure rate units, then:

$$R_a(t) = e^{\lambda_a t}$$
  $R_b(t) = e^{-\lambda_b t}$ 

And 
$$R_s(t) = \int_0^\infty R_s(t)dt = \frac{1}{\lambda_a} + \frac{1}{\lambda_b} - \frac{1}{\lambda_a + \lambda_b}$$

$$\theta_{s} = MTBF$$

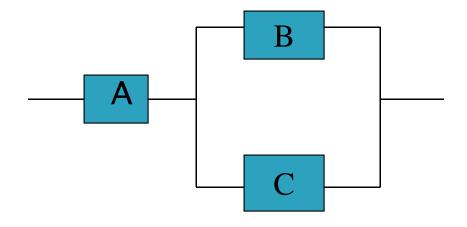
## Consider 3 components in parallel

- $R_s = 1 F_s$
- $F_a = 1 R_a$   $F_b = 1 R_b$   $F_c = 1 R_c$
- $R_s = 1 (1-R_a)(1-R_b)(1-R_c)$
- If components A, B and C are identical, then the reliability,

$$R_s = 1 - (1 - R)^3$$

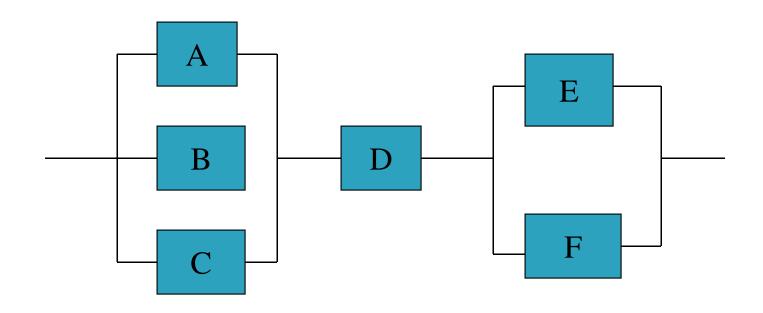
For a system with n identical components,

$$R_s = 1 - (1 - R)^n$$



$$R_s = R_A [R_B + R_C - R_B R_C]$$

$$Rs = [1-(1-R_A)(1-R_B)][1-(1-R_C)(1-R_D)]$$



 $Rs=[1-(1-R_A)(1-R_B)(1-R_C)][R_D] \times [R_E+R_F-(R_E)(R_F)]$ 

- For combined series-parallel network, first evaluate the parallel elements to obtain unit reliability
- Overall system reliability is determined by finding the product of all series reliability